A VIBRATION ELECTROMETER

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I. INTRODUCTION

The telephone and the vibration galvanometer [1] have long been used to detect very small alternating currents and voltages. The telephone is very sensitive in the range of frequencies from 500 to 3000 cycles, but has the disadvantage of responding to harmonics as well as to the fundamental. The vibration galvanometers are relatively insensitive to the harmonics, and are much more sensitive at the lower frequencies than is a telephone.

The sensitiveness of an instrument may be defined in terms of the *voltage* which must be applied to give unit deflection, or it may be defined in terms of the *current* which will give unit deflection.[2] In order that either a telephone or a vibration galvanometer shall be very sensitive to an alternating *current*, it must be con-

[1] The most important types of vibration galvanometers, the range of frequencies for which they are usually used, and the place where they are first described are as follows:

Type	Range (cycles)	Reference.
Rubens	50–200	Wied. Ann., **56**, p. 27; 1895.
Wien	50–1,000	Ann. d. Phys., **309**, p. 425; 1901.
Campbell	50–1,000	Phil. Mag., [6] **14**, p. 494; 1907.
Duddell	300–3,000	Phil. Mag., [6] **18**, p. 168; 1909.
Drysdale	20–200	Electr., **69**, p. 939; 1912.

[2] For a discussion of this, see F. Wenner, A theoretical and experimental study of the vibration galvanometer, this bulletin, **6**, p. 347, Reprint 134, 1909.

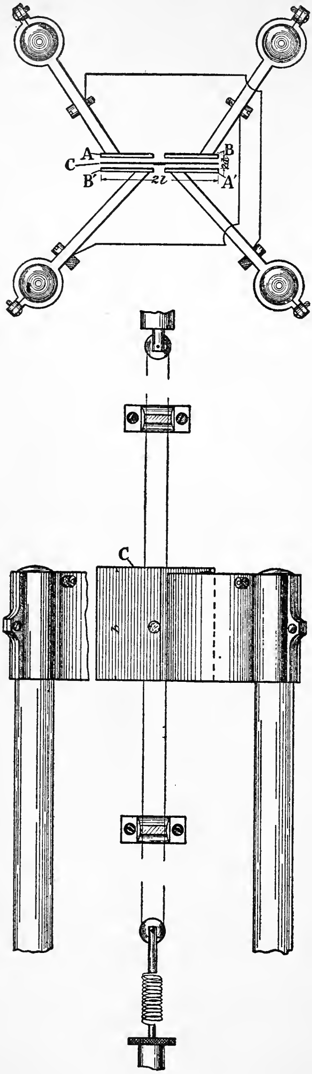

FIG. 1.—*Plan and elevation of the vibration electrometer*

structed of very fine wire. A limit is soon reached in this direction, due to the difficulties in making and handling very fine wire. A vibration electrometer will detect very much smaller alternating currents than either of the above instruments. Such an instrument has already been described by Greinacher,[3] although his description did not appear until after the instrument here described had been constructed. He adapted a Wulf electrometer, using a transformer in connection with the instrument.

The instrument here described is a modification of a quadrant electrometer. The need for it arose in connection with the measurements of very small capacities at low frequencies. By means of it, capacities of the value of a thousandth of a microfarad have been measured at 50 cycles with an accuracy about 10 times greater than can be obtained by any vibration galvanometer in this laboratory. For smaller capacities, the advantage is still greater. However, it is useful only when the impedance of the bridge arms is very high, so that the current which flows through them is very small. Also it can not be used at frequencies much above 100 cycles on account of the moment of inertia of the vane.

[3] Phys. Zs., **13**, pp. 388, 433; 1912.

As the design is of such form as to make a mathematical treatment of its behavior rather simple, the equations governing its operation have been worked out in some detail. Following this are some experimental results to show the agreement between theory and practice, together with some hints in the practical use of the instrument.

II. DESCRIPTION OF INSTRUMENT

The instrument consists of four metal plates A, A_1, B, and B_1, diagonally opposite plates being connected as shown in Fig. 1. Between these a light aluminum vane C is supported by means of a bifilar suspension. This vane is free to vibrate about a vertical axis. The plates correspond to the quadrants of a quadrant electrometer, while the vane corresponds to the needle. If an electrostatic charge is given to the vane, and an alternating electromotive force is applied to the plates, the vane will be forced to vibrate in the period of the applied electromotive force. If the natural period of the suspended system is identical with the period of the applied electromotive force, then the amplitude of vibration is largely increased. The natural period can be varied by changing the length of the bifilar suspension, by varying the distance between the suspensions, or by altering the tension on the suspensions. When in resonance, the amplitude will depend upon the damping, and as air damping is a large part of the total damping, the whole instrument is placed under a bell jar, from which the air can be exhausted.

III. THEORY OF THE INSTRUMENT

As the instrument is a modification of the quadrant electrometer, the theory will be much the same. However, the form of the instrument is such that calculations are somewhat simplified, and more exact results can be obtained.

The capacity of two rectangular planes inclined at an angle is given by Rosa and Dorsey [4] as

$$C=\frac{bl}{4\pi d_1}\left[1-\frac{1}{2}\frac{d_2-d_1}{d_1}+\frac{1}{3}\left(\frac{d_2-d_1}{d_1}\right)^2-\cdots\cdots\right]$$

providing edge corrections can be neglected.

[4] This Bulletin, **3**, p. 486, 1907, Reprint No. 65.

Where b is the breadth of the plates, l their length, d_2 the greatest distance between them, and d_1 the least distance.

$$\text{Putting } \sin\theta = \frac{d_2 - d_1}{l}$$

$$C = \frac{bl}{4\pi d_1}\left[1 - \frac{1}{2}\frac{l\sin\theta}{d_1} + \frac{1}{3}\frac{l^2\sin^2\theta}{d_1^2} - \ldots\ldots\right] \tag{1}$$

If d_1 remains constant while d_2 diminishes, it can be shown that formula (1) holds when d_2 becomes less than d_1, thus making θ negative. Hence we shall drop the subscript to d_1, writing it d.

If one of the plates vibrates in such a manner that d and l remain constant, while θ is a known function of the time, the capacity at any instant is given by equation (1). While, in this instrument, the condition that d should remain constant is fulfilled only when there is an infinitely small distance between A and B on the one side and A_1 and B_1 on the other, yet the approximation is very close with the distances used.

From Fig. 1 it is seen that there are three insulated conductors $A - A_1$, $B - B_1$, and C, so that six coefficients of capacity are necessary to determine the charges upon the plates when known potentials are applied to them.

If q_1 is the charge upon $A - A_1$ and E_1 its potential
If q_2 is the charge upon $B - B_1$ and E_2 its potential
If q_3 is the charge upon C and E_3 its potential

then

$$\left.\begin{aligned} q_1 &= C_{11}E_1 + C_{12}E_2 + C_{13}E_3 \\ q_2 &= C_{12}E_1 + C_{22}E_2 + C_{23}E_3 \\ q_3 &= C_{13}E_1 + C_{23}E_2 + C_{33}E_3 \end{aligned}\right\} \tag{2}$$

where C_{11}, C_{12}, etc., are the coefficients of capacity between the three systems.

From equation (1), it follows that

$$\left.\begin{aligned}
C_{11}&=K_1+\frac{bl}{2\pi d}\left(1+\frac{1}{2}\frac{l\sin\theta}{d}+\frac{1}{3}\frac{l^2\sin^2\theta}{d^2}+\cdots\right)\\
C_{22}&=K_2+\frac{bl}{2\pi d}\left(1-\frac{1}{2}\frac{l\sin\theta}{d}+\frac{1}{3}\frac{l^2\sin^2\theta}{d^2}-\cdots\right)\\
C_{33}&=K_3+\frac{bl}{\pi d}\left(1+\frac{1}{3}\frac{l^2\sin^2\theta}{d^2}+\cdots\cdots\cdots\right)\\
C_{12}&=-K\\
C_{13}&=-\frac{bl}{2\pi d}\left(1+\frac{1}{2}\frac{l\sin\theta}{d}+\frac{1}{3}\frac{l^2\sin^2\theta}{d^2}+\cdots\right)\\
C_{23}&=-\frac{bl}{2\pi d}\left(1-\frac{1}{2}\frac{l\sin\theta}{d}+\frac{1}{3}\frac{l^2\sin^2\theta}{d^2}-\cdots\right)
\end{aligned}\right\}\qquad(3)$$

Where b, l, d, and θ have the values previously given, and K, K_1, K_2, and K_3 are constants

If W is the total potential energy of the system at any instant, then

$$W=\frac{1}{2}C_{11}E_1^2+\frac{1}{2}C_{22}E_2^2+\frac{1}{2}C_{33}E^2_3+C_{12}E_1E_2+C_{13}E_1E_3+C_{23}E_2E_3 \quad (4)$$

If the torque at an angle θ is M

$$M=\frac{dW}{d\theta}$$

Substituting values from (3) in (4) and taking the derivative.

$$\begin{aligned}
M=\frac{bl}{4\pi d}\Bigg\{&E_1^2\left(\frac{1}{2}\frac{l\cos\theta}{d}+\frac{2}{3}\frac{l^2\sin\theta\cos\theta}{d^2}+\cdots\right)\\
&+E_2^2\left(-\frac{1}{2}\frac{l\cos\theta}{d}+\frac{2}{3}\frac{l^2\sin\theta\cos\theta}{d^2}-\cdots\right)\\
&+E_3^2\left(+\frac{4}{3}\frac{l^2\sin\theta\cos\theta}{d^2}+\cdots\right)\\
&+E_1E_3\left(-\frac{l\cos\theta}{d}-\frac{4}{3}\frac{l^2\sin\theta\cos\theta}{d^2}-\cdots\right)\\
&+E_2E_3\left(\frac{l\cos\theta}{d}-\frac{4}{3}\frac{l^2\sin\theta\cos\theta}{d^2}+\cdots\right)\Bigg\}
\end{aligned}$$

If now $E_1 = -E_2 = E_0 \cos pt.$, where E_0 is the maximum value of the alternating electromotive force, p is 2π times its period, and t the time.

$$M = \frac{bl}{4\pi d}\left\{\frac{4}{3}\frac{l^2 \sin\theta\cos\theta}{d^2}(E_0^2\cos^2 pt + E_3^2) - \frac{2l\cos\theta}{d}E_0E_3\cos pt.\right\} \quad (5)$$

If θ is small, then for a first approximation, $\sin\theta = \theta$ and $\cos\theta = 1$.

Also if E_3^2 is large relative to E_0, equation (5) reduces to

$$M = \frac{bl}{4\pi d}\left\{\frac{4}{3}\frac{E_3^2 l^2\theta}{d^2} - \frac{2lE_0E_3}{d}\cos pt\right\}$$

$$= \frac{1}{3}\frac{bl^3E_3^2}{\pi d^3}\theta - \frac{1}{2}\frac{bl^2E_0E_3}{\pi d^2}\cos pt \quad (6)$$

In any system free to move around an axis, the sum of all the torques at any instant must equal zero.

Hence

$$K\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + V\theta = \frac{1}{3}\frac{bl^3E_3^2}{\pi d^3}\theta - \frac{1}{2}\frac{bl^2E_0E_3}{\pi d^2}\cos pt \quad (7)$$

Where K is the moment of inertia of the rotating system,
D is the damping coefficient,
V is the restoring couple of the suspension.
and θ is the angle of rotation.

The complete solution of this equation is

$$\theta = C_1e^{(-\alpha+\beta)t} + C_2e^{(-\alpha-\beta)t} + \delta\cos(pt-\gamma) \quad (8)$$

Where

$$\alpha = \frac{D}{2K}; \quad (9)$$

$$\beta = \frac{1}{2K}\sqrt{D^2 - 4KV + \frac{4}{3}\frac{Kbl^3E_3^2}{\pi d^3}} \quad (10)$$

$$\delta = \frac{1}{2}\frac{bl^2E_0E_3}{\pi d^2\sqrt{D^2p^2 + \left(V - \frac{1}{3}\frac{bl^3E_3^2}{\pi d^3} - Kp^2\right)^2}} \quad (11)$$

$$\gamma = \tan^{-1}\frac{-Dp}{V - \frac{1}{3}\frac{bl^3E_3^2}{\pi d^3} - Kp^2} \quad (12)$$

The first two terms of the second member of equation (8) diminish with the time. They are of importance for only a short time after closing or opening the circuit. Hence, if the instrument is kept on closed circuit they may be neglected.

In the use of the instrument we are desirous that the value of δ, which is the amplitude of vibration, should be a maximum for a given value of E_0. In other words, if the instrument is used on an alternating-current bridge, a slight lack of balance should produce a relatively large vibration of the instrument. An examination of expression (11) shows that for a maximum value of δ, D, the damping coefficient, should be small. This damping is of two parts—the friction and molecular loss in the suspension wires, and the damping due to air friction. In this instrument the air damping is a very large part of the total loss, and this can be greatly diminished by putting the instrument under a bell jar and exhausting the air. However, as will be pointed out later, it may not be desirable to reduce the damping to as low a point as possible.

If the expression (11) is differentiated with respect to E_3, assuming all the other quantities constant, and the result equated to zero, the resulting equation serves to determine the value of E_3 which will make δ a maximum. This value of E_3 is

$$E_3 = \sqrt[4]{K^2p^4 + (D^2 - 2VK)p^2 + V^2}\sqrt{\frac{3\pi d^3}{bl^3}} \tag{13}$$

In the same way, by differentiating (11) with respect to p, the value of p for a maximum δ is

$$p^2{}_m = \frac{V - \dfrac{bl^3E_3^2}{3\pi d^3} - \dfrac{D^2}{2K}}{K} \tag{14}$$

Hence, we see from (14) that if E_3 is increased p is decreased to obtain a maximum value of δ. but V, the restoring couple, must always be larger than

$$\frac{bl^3E_3^2}{3\pi d^3} + \frac{D^2}{2K}$$

since the frequency must be greater than zero. By substituting (14) in (11) the value of δ, when p is adjusted for a maximum value is

$$\delta_m = \frac{1}{2}\frac{bl^2E_0E_3\sqrt{K}}{\pi d^2D\sqrt{V-\frac{bl^3E_3^2}{3\pi d^3}-\frac{D^2}{4K}}} = \frac{bl^2E_0E_3\sqrt{K}}{2\pi d^2D\sqrt{Kp^2_m+\frac{D^2}{4K}}} \tag{15}$$

From this it is seen that an increase of E_3 will cause an increase of δ, both by increasing the numerator and decreasing the denominator until $V=\frac{bl^3E_3^2}{3\pi d^3}+\frac{D^2}{4K}$ when δ becomes infinite. Thus, we see that if p and E_3 can both be varied, increasing E_3, the voltage on the vane, decreases p the frequency for a maximum amplitude, and this maximum amplitude δ_m increases more rapidly than E_3. If E_3 is sufficiently large, $p=0$ and $\delta_m=\infty$, though as E_3 increases p becomes zero before δ_m becomes infinite. Hence when E_3 is too large, the vane, if slightly displaced, deflects until it strikes the plates and the electrostatic forces which tend to keep it in this position are greater than the restoring forces of the suspension.

The conditions for maximum amplitude are, therefore, small damping, a large but not excessive voltage on the vane, and the frequency of the applied voltage adjusted until the maximum amplitude is obtained. These conditions are in addition to those which are imposed by the instrument itself, such as the size and mass of the vane, the distance apart of the plates, and the torsional rigidity of the suspension. A discussion of the effect of these would follow the methods already indicated.

Next to securing a large amplitude of vibration for a small electromotive force impressed upon the plates comes the question of sharpness of tuning. It may be that a vibrating instrument will give a very large amplitude for a particular value of the frequency, yet the amplitude will decrease so rapidly with changes in frequency that it is a very difficult instrument with which to work. This is due to the fact that no source of alternating electromotive force is entirely free from fluctuations in frequency.

The sharpness of tuning S may be defined as the reciprocal of the resonance range R, where the resonance range is the difference, in proportional parts, between the frequency for maximum ampli-

tude, p_m, and the frequency $p_{\frac{m}{2}}$ which will reduce the amplitude to one-half of the maximum. Or in symbols

$$S = \frac{1}{R} = \frac{p_m}{p_m - p_{\frac{m}{2}}} \tag{16}$$

If the value of δ given in (11) is put equal to one-half of the maximum value of δ given in (15), and this equation solved for p^2 (which then is $p^2_{\frac{m}{2}}$)

$$p^2_{\frac{m}{2}} = p^2_m \pm \frac{D}{2K^2}\sqrt{12K^2 p_m^2 + 3D^2}$$

Therefore

$$p_m - p_{\frac{m}{2}} = \pm \frac{D\sqrt{12K^2 p_m^2 + 3D^2}}{2K^2\left(p_m + p_{\frac{m}{2}}\right)}$$

If the resonance range is small relative to $2p_m$, $p_m + p_{\frac{m}{2}} = 2p_m$ approximately.

$$\therefore R = \frac{D\sqrt{12K^2 p_m^2 + 3D^2}}{4K^2 p_m^2} \tag{17}$$

When used as a sensitive instrument D is small relative to Kp_m. Then

$$R = \frac{D\sqrt{3}}{2Kp_m}$$

Hence the resonance range is proportional to the damping and inversely proportional to the moment of inertia of the moving system. Stated otherwise, the sharpness of tuning increases as the damping decreases or as the moment of inertia increases. By reference to equation (11) we see that whatever the frequency, a decrease in the damping will increase the amplitude, though it is only in the neighborhood of resonance that this has an appreciable effect.

It may be desirable to know what current is necessary to operate such an instrument. This is somewhat complicated by the fact that the current in the two wires leading to the two plates is not the same at any instant, though the integral value is the same.

The current in the wire leading to $A-A_1$ can be found by differentiating q_1 in equations (2) with respect to the time. In case E_3 is large relative to E_0 and hence is large relative to E_1 and E_2, and to their derivatives when the frequency is low, we have as a first approximation

$$i_1 = \frac{dq_1}{dt} = E_3 \frac{dc_{13}}{dt} = -E_3 \frac{bl}{2\pi d}\left(\frac{1}{2}\frac{l}{d} + \frac{2}{3}\frac{l^2\theta}{d^2} + \cdots\right)\frac{d\theta}{dt}$$

As θ is a small quantity, we can neglect the second term of this expression. Then, since $\theta = \delta \cos(pt - \gamma)$, it follows that

$$\frac{d\theta}{dt} = -\delta p \sin(pt - \gamma)$$

and

$$i_1 = E_3 \delta p \frac{bl^2}{4\pi d^2} \sin(pt - \gamma) \tag{18}$$

From this we see that δ, the amplitude of vibration of the vane, is directly proportional to the current. If values of δ and γ under the condition that δ is a maximum are substituted in (18),

$$i_1 = \frac{E_3^2 E_0 p_m b^2 l^4 K}{4\pi^2 d^4 (4p_m^2 K^2 + D^2)} \sin pt + \frac{E_3^2 E_0 p_m^2 b^2 l^4 K^2}{2\pi^2 d^4 D (4p_m^2 K^2 + D^2)} \cos pt. \tag{19}$$

From this we see that the in-phase component of the current (the coefficient of $\cos pt$) is inversely proportional to the damping. Hence as the damping is diminished the in-phase current, and therefore the power supplied to the moving system, increases for a given value of the emf., E_0, impressed on the plates.

The power, P, supplied to the moving system can be obtained from (19) and (15) in the form $P = \frac{\delta_m^2 p_m^2 D}{2}$. This shows that if δ_m is maintained constant, P decreases in the same ratio as D.

Under the assumptions which we have made, the current flowing in the conductor which leads to the plates $B-B_1$ is equal and of opposite sign to the current flowing to the plates $A-A_1$. If the neglected terms are included, this is not the case. This is most easily shown by determining the current that flows in the conductor leading to the vane. This is found from the value of q_3 in equation (2) by differentiating with respect to the time. Carrying out this process and substituting values, $i_3 = H \sin(2pt - \epsilon)$,

where H and ϵ are constants whose values can be easily evaluated. Since $i_1+i_2+i_3=0$, it follows that i_1 is not equal to $-i_2$.

The preceding discussion has been based upon the assumption that the instrument is on closed circuit. In actual use the current is changed by adjusting the bridge for which the electrometer is serving as a detecting instrument, and it is desirable to have the deflection reach a steady condition as soon as possible. In order to determine this, it will be necessary to examine the values of α and β in equation 8. If we neglect for the moment the periodic term $\delta \cos (pt-\gamma)$ the remaining portion θ_1 may be written in the form

$$\theta_1=e^{-\alpha t}\left(C_1e^{\beta t}+C_2e^{-\beta t}\right)$$

By substituting the value of p_m from equation (14) in (12),

$$\beta=\frac{1}{2K}\sqrt{-4p_m^2K^2-D^2}$$

Hence β is always imaginary under the condition of maximum deflection. In this case θ_1 is periodic, having an amplitude $Re^{-\alpha t}$ and a period which is nearly equal to p_m when the damping is small. The constant R depends upon the initial conditions and can not be determined except as these are known. The rate at which the amplitude decreases with time depends upon α. Since $\alpha=\frac{D}{2K}$, the rate at which the exponental term will disappear will increase as the damping, D, increases. To be able to use the instrument satisfactorily, the damping must be large enough so that the deflection will reach its normal value within a few seconds after closing the circuit or making any change which will affect the flow of current through the instrument.

IV. EXPERIMENTAL VERIFICATION OF THE THEORY

The experimental work has been carried on with a view of testing the validity of the formulas developed in the theoretical work. A diagram of the set-up used in this work is shown in Fig. 2. Four condensers, C_1, C_2, C_3, and C_4, are arranged in the form of a Wheatstone's bridge, C_3 and C_4 being mica condensers of the same

nominal value, while C_1 and C_2 are variable air condensers. A small resistance r is connected in series with the mica condenser having the lowest phase difference and adjusted to make the phase difference of the mica condensers the same. To this is connected an alternating current voltage, whose frequency can be varied over quite wide limits by varying the speed of the generator. The voltage applied to the bridge is measured by a voltmeter V. The midpoints of the bridge are connected to the plates of the vibrating electrometer, while the vane of the instrument is connected to a battery whose voltage can be varied in steps of 40 volts.

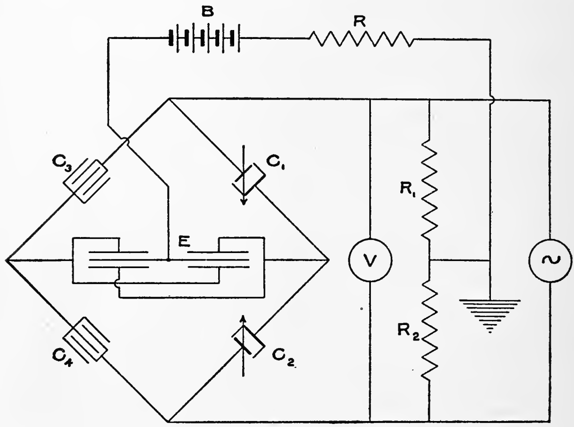

Fig 2.—*Diagram of the set-up for testing the vibration electrometer*

The opposite pole of the battery is connected to earth through a high resistance R. The value of R must be sufficiently large to prevent reactions between the bridge and the battery, but not so large as to be comparable with the leakage paths from the battery. In this investigation R has been kept at one megohm. The terminals of the bridge are connected to two equal resistances R_1 and R_2 in series, the midpoint of these resistances being grounded. Hence the midpoint of the bridge is also at earth potential at every instant.

Upon the vane of the electrometer there is mounted a small mirror. The image of a lamp filament formed by this mirror is

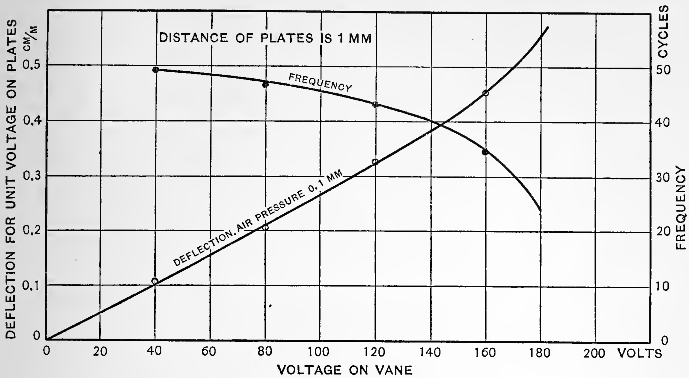

FIG. 3.—*Curves showing the effect of voltage on the vane upon the frequency at which resonance occurs and upon the deflection at resonance for one air pressure and for a distance between the plates of 1 mm*

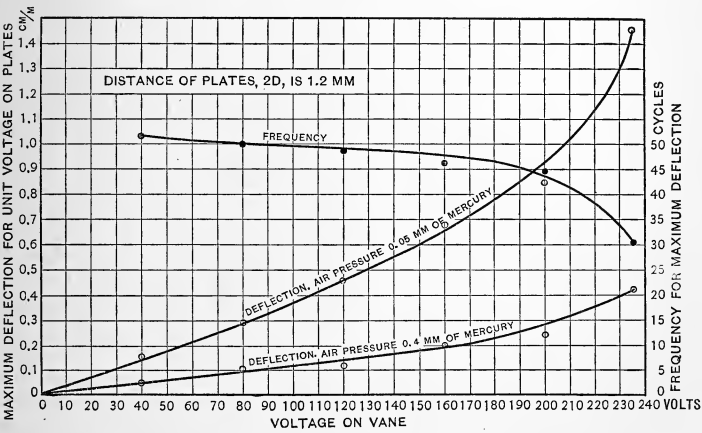

FIG. 4.—*Curves showing the effect of voltage on the vane upon the frequency at which resonance occurs and upon the deflection at resonance for different air pressures and for a distance between the plates of 1.2 mm*

viewed by a telescope in the eyepiece of which there is a graduated scale. For determining the sensitiveness and sharpness of tuning under any given conditions, C_1 and C_2 are adjusted until a balance is obtained; then C_1 is decreased until at the frequency of maximum sensitiveness the deflection is two divisions of the eyepiece scale. From the values of C_1, C_2, and the applied voltage the sensitiveness of the instrument can be computed. Then the frequency is varied on each side of the maximum until the deflection is reduced to one division. This gives the data for computing the sharpness of tuning.

In all of the experiments the size of the vane and of the plates has remained the same, viz, $l = 1$ *cm* and $b = 2$ *cm*. The distance, $2d$, between the plates is variable, and in some experiments the effect of varying this distance has been shown. Its value will be given in all cases.

In Fig. 3 is given a curve showing the maximum deflection for unit voltage on the plates with different voltages on the vane (the distance between the plates being 1 mm). In Fig. 4 are exactly similar curves where the distance between the plates is 1.2 mm. The air pressure in the bell jar surrounding the instrument is given for each curve. It will be noted in each case that while the curve is approximately a straight line for low voltages on the vane, yet as the voltage increases a point is reached where the deflection increases rapidly. With a distance of 1 mm between the plates the vane deflected so as to strike the plates when 200 volts were applied. In the discussion of equation (15) it was pointed out that such a condition would be reached. It was also pointed out that, as that condition is approached, the frequency will approach zero. In Figs. 3 and 4 the frequency is also plotted as a function of the voltage on the vane. It will be seen that the frequency begins to decrease markedly at the same time that the sensitiveness begins to increase.

In Fig. 5 are curves showing the sensitiveness with a distance of 2 mm between the plates. In this case the pressures have been reduced much lower than in the preceding cases, thus making the measurements much more difficult. For the range of voltages used the curves are straight lines within the limit of measurement.

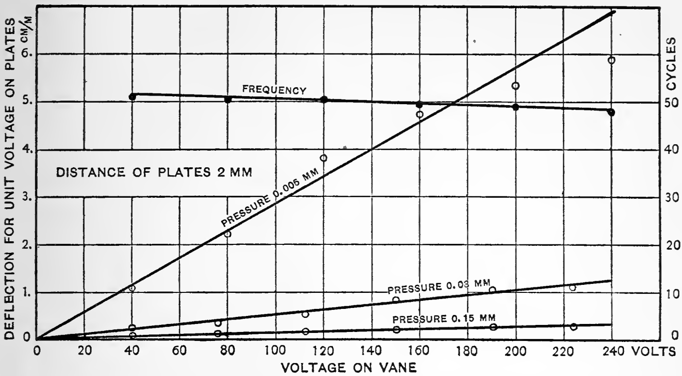

FIG. 5.—*Curves showing the effect of voltage on the vane upon the frequency at which resonance occurs and upon the deflection at resonance for different air pressures and for a distance between the plates of 2 mm*

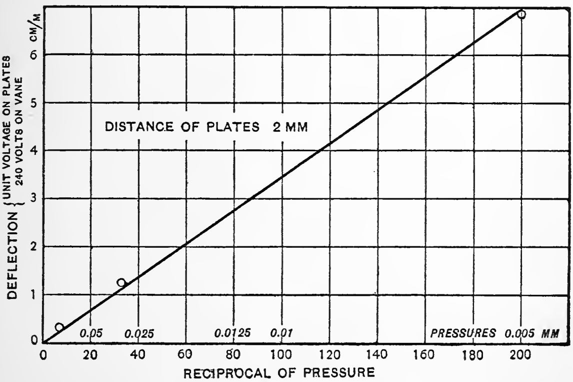

FIG. 6.—*Curve showing that the deflection is inversely proportional to the damping*

From the mean values of the deflection obtained from the curves of Fig. 5 are obtained values by means of which the curve of Fig. 6 is plotted. We may assume that the air damping of the vane is proportional to the air pressure.[5] Then when the damping due to the friction of the suspension is comparable to that due to the air, the curve will bend toward the horizontal. As there is no such tendency at a pressure of 0.005 mm of mercury, it is apparent that the damping due to the suspension is exceedingly small. In some preliminary experiments when the friction of the suspension over the bridge was rather large the damping due to the suspension was not negligible. This curve shows experimentally that which is derived theoretically in equation (15), viz, that the maximum amplitude is inversely proportional to the damping.

In equation (17) it is shown from theoretical considerations that the resonance range is proportional to the damping, provided $4K^2p_m^2$ is large relative to D^2. In Fig. 7 is given a curve showing the experimental relationship found. Within the experimental error the curve is a straight line.

In Fig. 8 are shown curves representing the deflection at different frequencies with different air pressures. It shows in a very marked way the increase in sensitiveness and the decrease in resonance range as the damping decreases.

The current which flows into the instrument can be determined from equation (18) by means of the data given in Fig. 8. From this we find that the current required to give a deflection of 1 cm at a meter's distance is approximately 10^{-9} ampere. As an observer can detect one one-hundredth of this deflection, the current which will produce a noticeable deflection is only 10^{-11} ampere.

V. SUMMARY

An electrostatic instrument, called a vibration electrometer, is described. It is capable of detecting alternating currents of low frequency having a value as small as 10^{-11} ampere. The theory of the instrument is developed mathematically and the conclusions verified by experiment. The important conclusions are as follows:

1. For any given adjustment of the instrument, the frequency at which maximum deflection is obtained depends upon the poten-

[5] See Hogg, Friction in Gases at Low Pressures. Phil. Mag. [6], 19, p 376: 1910.

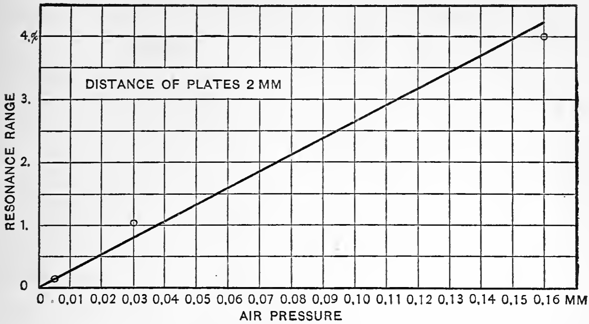

FIG. 7.—*Curve showing that the resonance range is proportional to the damping*

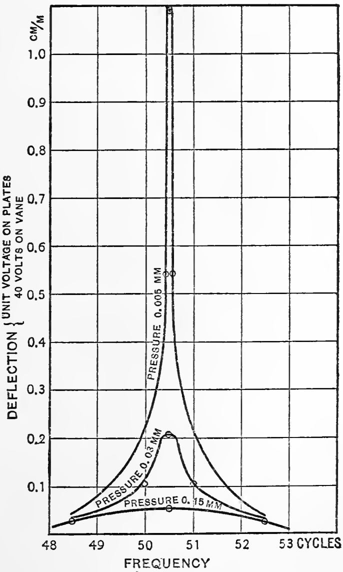

FIG. 8.—*Curves showing the increase in sensitiveness and decrease in resonance range as the damping is decreased*

tial of the vane. As the potential of the vane is increased, the frequency at which maximum deflection is obtained is decreased.

2. When the voltage on the vane is increased, the deflection for a given voltage on the plates increases more rapidly than the first power of the voltage. The sensitivity can not be increased indefinitely in this way, since the frequency will become zero before the sensitiveness becomes infinite.

3. The deflection is inversely proportional to the damping. It is shown experimentally that the damping due to a well-constructed suspension is exceedingly small.

4. As the damping is decreased, the range of frequencies over which the instrument can be used is greatly diminished. Hence it is not important to decrease the damping beyond a reasonable point.

5. Upon closing the circuit or otherwise changing the current through the electrometer, some time is required before the amplitude of vibration becomes constant. This time will be increased as the damping is decreased.

6. The power required to give unit deflection when the applied emf is in resonance with the instrument decreases in the same ratio as the damping.

WASHINGTON, September 29, 1914.